Math 10B with Professor Stankova
Worksheet, Discussion #10; Tuesday, 2/20/2018
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1 Bayes Theorem and Independence

1.1 Concepts

1. We use **Bayes theorem** when we want to find the probability of A given B but we are told the opposite probability, the probability of B given A. There are several forms of Bayes Theorem as follows:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{1}{1 + \frac{P(B|\bar{A})P(\bar{A})}{P(B|A)P(A)}}.$$

In order to discern which form to use, look at the information you are given. If you are told $P(B|A)$ as well as $P(B|\bar{A})$, use the latter two methods but if you are only told $P(B)$, then use the first form.

We say that two events A, B are **independent** if $P(A \cap B) = P(A)P(B)$.

1.2 Examples

2. When rolling a fair 6-sided die, are the events A that the number rolled is greater than or equal to 3, and B that the number rolled is odd, independent?

Solution: We just need to check $P(A \cap B) = P(A)P(B)$. On the left side, the probability is $\frac{2}{6}$ from having $3, 5$, and $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{2}$ so indeed $P(A \cap B) = P(A)P(B)$. So they are independent.

3. There are 10 red and 10 blue balls in a bag. Someone randomly picks out a ball and then places it back and puts 10 more balls of that color into the bag. Then you draw a ball. What is the probability that the 10 balls added were red, given that you drew out a red ball?

Solution: We use Bayes Theorem to get

$$P(AddRed|DrawRed) = \frac{1}{1 + \frac{P(DrawRed|AddBlue)P(AddBlue)}{P(DrawRed|AddRed)P(AddRed)}}$$
$$= \frac{1}{1 + \frac{10/30 \cdot 1/2}{20/30 \cdot 1/2}} = \frac{1}{1 + 1/2} = \frac{2}{3}.$$

1.3 Problems

4. **TRUE** False If A, B are mutual exclusive events that are independent, then $P(A) = 0$ or $P(B) = 0$.

> **Solution:** If A, B are mutually exclusive, then $A \cap B = \emptyset$. Then if they are independent, then $P(A \cap B) = 0 = P(A)P(B)$ so $P(A) = 0$ or $P(B) = 0$.

5. True **FALSE** If A, B are independent events and B, C are independent, then A, C are independent.

> **Solution:** We can take A and C to be the same event.

6. I roll two die. Are the events that the first die roll is a 1 and that the sum of the two dice is a 7 independent?

> **Solution:** Let A be the event that the first die roll is a 1 and let B be the event that the sum of the two dice is a 7. Then we can compute $P(A) = P(B) = 1/6$ and $P(A \cap B) = 1/36$, so they are independent.

7. What is the probability that a family with two kids has two boys if you know at least one is a boy?

> **Solution:** This is a tricky problem. Let A be the event that you have at least one boy and let B be the event that you have two boys. Then by Bayes, we have that
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> $$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{1 \cdot 1/4}{3/4} = \frac{1}{3}.$$